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TRANSITION STATE IN PATTERNS OF HISTORY

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ABSTRACT

A review of literature shows that a diaspora of natural scientists interested in history has been forming for some time around the legacy of Rashevsky, Richardson, and Tilly. Analytical history of Bertrand Roehmer and Tony Syme, as well as the study of patterns of military conflicts by Peter Brecke, have a potential of becoming centers of the "naturalization" of historical research.

The existing formal approaches to complex systems create a conundrum of the use of closed mathematical structures for representing open irreversible systems. Pattern Theory (Ulf Grenander) is suggested as another entry in the inventory of methods in this area.

Pattern Theory, with its atomistic realism and preservation of semantics, is uniquely positioned for developing a general representation of Very Complex Open Systems, such as life, mind, and society. The focus of the application of Pattern Theory to history is transition state characterized by its irregularity and, therefore, instability.

A possibility to treat a segment of history as a quasi-chemical structural transformation through alternating stable and irregular states is illustrated on the example of the expedition of Darius against the Scyths.

CONTENTS

1. Preamble

The divide between the two cultures—sciences and humanities—appears to be slowly closing due to two trends. One is the influence of the pervasive science and technology on the creative process in humanities. An installation of the artist Janine Antoni who weaves a recorded pattern of her own Rapid Eye Movements on a traditional loom is an example. The other trend is the unstoppable spread of sciences, enhanced by mathematics and computers, over the traditional areas of humanities. Social sciences seem to be an emerging Isthmus of Panama between the two previously separated continents. Compare three quotations, one of a sociologist, the second of a physicist, and the third of a fiction writer.

> I should like to deal with the promise of social science in terms of three prospects which seem to me both possible and desirable for the twenty-first century: the epistemological reunification of the so-called two cultures, that of science and the humanities; the organizational reunification and redivision of the social sciences; and the assumption by social science of centrality in the world of

> knowledge (Wallerstein, 1998).

> Today we are becoming more and more conscious of the fact that on all levels, from elementary particles to cosmology, randomness and irreversibility play an ever-increasing role. Science is rediscovering time. This obviously introduces a new dimension into the old problem of the two cultures, science and the humanities (Prigogine, 1983) .

> But Denise left the kitchen and took the plate to Alfred, for whom the problem of existence was this: that, in manner of a wheat seedling thrusting itself up out of the earth, the world moved forward in time by adding cell after cell to its leading edge, piling moment on moment, and to grasp the world even in its freshest, youngest moment provided no guarantee that you'd be able to grasp it again a moment later (Franzen, 2001, p. 66).

In the same book Jonathan Franzen uses adjective "Cherenkov blue," comparing an eye color to a physical effect that can be observed only in some nuclear reactors.

The above excerpts shall usher us into the area that has been attracting attention of philosophers since times immemorial and physicists, mathematicians, and computer scientists for decades: history. What can hard sciences say about human history? This time it is a take of a chemist.

In his *The Rise and Fall of the Great Powers* Paul Kennedy (1987) discussed the reasons why no great power in the last 500 years of history was able to maintain its status. Neither was any great empire of the earlier times, although at the peak of power the decline seemed for most observers unthinkable. In his subsequent book Kennedy (1993) reiterated and emphasized numerous tensions that could threaten the current American status.

Kennedy's earlier book was written in before the collapse of the USSR and the later one before the war on terrorism and the rise of anti-Americanism. Nevertheless the aforementioned conclusion about the transient character of economic power and military domination seems to be a *regularity* of history. If history comes as a surprise, then one day we might be surprised by a great power, remaining indefinitely unchallenged, as if history ended its regular course, in accordance with Francis Fukuyama's (1992) conclusions. If history had the invariance analyzed by Kennedy and obvious from any textbook of history, the *pattern* of the rise and decline would persist. The problem with this reasoning is that we have no means to supply any facts whether in support or in refutation because they reside in the future. In the absence of facts, a theory is a substitute, but there is no such universally accepted theory.

In spite of an accumulated library of patterns, warning signs, and wise predictions, history always surprises the contemporaries, requiring a *post factum* explanation. The main reason for this Cassandra Effect is the impossibility to tell **when** the next turn comes.

Paul Kennedy, emphasizing the difficulty of predicting the future, presented a series of arguments pro and contra the thesis that USA was entering a period of decline as a great power. The question is: what is the relation between the regularity and surprise in history? What is regularity (law, invariance), surprise (irregularity, novelty, invention), and what is a rationale for a particular timing of a historical event?

It may be too early to expect satisfying answers to these questions, but it might be the right time to ask in what direction should we look for them.

2. The intent of the paper

It would be possible to discuss the above example in search for a consensus if there were some theoretical foundations of history. At a certain level of abstraction, the theory would be invariant regarding a substitution of Roman Empire for USA or the second Gulf War for the war of Persian king Darius the Great with the Scyths. No historian, however, would sacrifice the obvious differences.

If we remove the terms (semantics) from consideration, what remains is relations (syntax), which is all that most mathematics is about. A search for such foundations by means of natural sciences, which unlike humanities are based on consensus rather than on institutionalized dissent (compare with dissent-based philosophy, Collins, 1998), has its own history in the context of the study of *complex systems*. They will be further abbreviated as LMS—life, mind, and society—in order to avoid the difficult problem of defining complexity where no consensus exists.

The purpose of this paper is to informally present Pattern Theory (Grenander, 1978-2003) as a possible component in scientific study of LMS and history in particular. I am not a mathematician to whose natural habitat Pattern Theory would belong but a chemist who intuitively perceives it as a kind of meta-chemistry, i.e., theory of atomistic structures.

My first suspicion that chemistry is a kind of a metaphor for history goes back to the 1970s, when I, together with many intellectuals in the former USSR, saw the Communist system as self-destructible and doomed to collapse. The question that nobody could answer—as nobody can answer the questions about current great and small powers—was: **when**? Today practically all professionals in the field of history and political science express their *surprise* that it came so *soon*. For about a decade after 1917, when the Bolsheviks came to power, the observers were *surprised* that it had lasted so *long*.

Chemistry studies events in systems with extremely high and incompressible complexity. It possesses some uncommon tools to answer questions pertaining both to the nature of possible events and their **timing**. A chemist sees time through bifocal glasses: one lens for long distance in time and another for a close-up. Remarkably, it reminds the vision of a historian of the French *Annales* school, which distinguishes between *la longue duree* (Braudel**,** 1992) and the short run of smaller scale *events*. This type of historian regards the totality of all factors, such as economics, geography, beliefs, etc., and pays attention to numbers.

The outcome of a chemical transformation *in the long run* is not the same as observed with a stopwatch. The former inexorably goes toward equilibrium, while the latter is the result of the **fastest** among concurrent transformations. Catalysis can selectively speed up some transformations. By using the entirely catalytic and therefore fast biochemistry, the living organism escapes equilibrium but pays for that by the absence of anything comparable with the historical *longue duree*: the "great power" of an individual organism always collapses.

Bertalanffy (1968) defined *system* as a set of units with relationships between them, which is the same as to say that a "systemic" system can be mapped on a mathematical system. An important novelty was Bertalanffy's emphasis on open systems and steady states instead of equilibrium.

Mathematical systems and structures, however, operate under the axiom of closure, which is sometimes regarded as part of the definition of operation. Although renormalization can change the base set in an orderly manner, the new set is closed, too. I am not aware of any inquest into the possibility of describing an open system in terms of a strictly formalized and closed one. The concept of and open *system* represented by a closed one, however, is just one possible liability of the traditional systemic approach. The other one, not too often discussed, encumbers the very term *theory*. What is theory? There are theories of philosophical grandeur, aspiring the ultimate generality, and there are working theories seeking empirical practicality and subject to experiment.

The additional purpose of this paper follows from my belief that a scientific *theory* of the second type, as it is most widely understood, has to satisfy a criterion of *realism*, i.e., being *recognizable* by an empiricist as relevant for the appropriate domain of a particular hands-on science. Thus, any meta-chemical construct should conform to a chemist's view of chemical phenomena and any meta-theory of history should not be so elevated above the daily craft of historians that the names of persons and circumstances of events would be obliterated by symbolic formalism. In other words, the theory has to fill up the space between the roof gardens of abstract theory and the basement storage of the facts. The contraposition of theory and practice is sometimes expressed in terms of a fissure between syntax and semantics.

This paper borrows important ideas from the manuscript *History as Points and Lines* by the author and Ulf Grenander. The entire project of pattern history was initiated by Ulf Grenander in 1994 and most important ideas belong to him. My own contribution is limited to the chemical angle of view, use of the concept of transition state, and the idea of an open (Heraclitean) formal system, in contrast to the closed (Aristotelian) system (Tarnopolsky, 2003).

The following short and selective *historical* overview is intended to help illuminate the search for mathematics of history as well as the epistemic place of Pattern Theory in among other approaches.

3. Historical sketch

Leo Tolstoy filled up the volumes of his *War and Peace* with hundreds of names, situations, and events, recreating the atmosphere of a society in the process of a historical transformation from peace to war and back. He did it in terms of observables: name, facial expression, movement, utterance, location, thought, etc. He concluded his epic with a chapter of a contrasting nature where he outlined his theory of history, using images and metaphors of calculus and chemistry to present history and war as what we would call now a dynamical system of statistical physics. It could well be described by differential equations as well as intensive (will to fight) and extensive (army size) parameters. On Tolstoy's mathematics, see Vitányi .

After Leo Tolstoy, one of the most recent—and gripping—attempts of humanities to paint a historical picture with colors borrowed from sciences belongs to philosopher **Manuel De Landa** (1997), who, for example, unites the origin of minerals, emergence of skeletons in biological evolution, and the rise of the cities under the same metaphor of mineralization. The close relation between metaphor and what is intuitively understood by the term *pattern* is palpable. This line leads back to **Michel Foucault** (1970) whose thinking was clearly mathematical but the language metaphoric and artistic. On metaphor in historiography—and the complex picture of the modern philosophy of history—see Ankersmit (1994).

In 1968, one hundred years after *War and Peace*, **Nicolas Rashevsky** published his *Looking at History through Mathe*matics (Rashevsky, 1968). He defined the purpose of his book as a collection of illustrations "how mathematical reasoning *could in principle* be used in attempted explanations of *some* historical phenomena." Among them, imitative behavior and the development of aggressiveness, beliefs and prejudices, effect of the shoreline on cultural development, role of individual (**without mentioning** Leo Tolstoy), and some other aspects of "long history." In his earlier "*Mathematical Biology of Social Behavior*" (Rashevsky, 1959), mathematics of history was a topic of an appendix, with such subjects as mathematics of political freedom.

In modern terms, what Rashevsky applied to history could be called population dynamics started by Lotka (1956|1925) and Volterra (1931), who were inspired, by the way, by chemical kinetics. Rashevsky focused on the populations of *memes*, although Richard Dawkins (1989), who invented the term, **did not mention** Rashevsky in his book. For example, "militarism" and "pacifism," "laziness" and "activity," are competing pairs of behavior reinforced by mutual imitation, i.e., replication, and inhibition for the sake of consistency of mentality. The main mathematical apparatus applied—and employed also in chemistry and artificial life—is differential equations.

Rashevsky gradually became dissatisfied with the fragmentary descriptions of various aspects of LMS, including his own contributions. He came to the conclusion that physics in metric space may give a lot of details about life but not the entire view of the living system. He formulated a cardinally different idea of "relational" biology and sociology as an opposite of the "metric" and fragmentary physical view of the complex systems. For Rashevsky, therefore, the term "mathematics" in applications to LMS has two meanings: (1) the down-to-earth calculations along a *physical* model developing as a time series of states and (2) the *geometrical* (in Felix Klein's sense) view of the system. He believed that the two should better be "objects of parallel studies."

As an example, a whole array of real structures, from the movement of a bird to composing a love sonnet can be mapped on a directional graph of realtions. He specified the edge of such a graph as "immediate precedence" or "immediate causation." Similarly, the movement of a paramecia toward food is a relation, which is fully preserved in humans, in spite if the evolutionary gap. Rashevsky stated that a complex (*i.e.*, LMS) system should be described in terms of *patterns* defined as families of bijective mappings of one system on another (homeomorphisms). In his *Mathematical Biophysics* , Rashevsky saw "the organism as a set of mappings, categories, and equivalences." (Rashevsky, 1960, Vol.2, p. 390).

Rashevsky's abstract ideas were further developed by his student **Robert Rosen** (1985-2000) whose major interests comprised general theory of natural systems, including LMS. In particular, Rosen was interested in anticipatory systems **where the change of state depends on future circumstances**, which returns us to the starter example. For such systems a model is a must. It was the interaction through an interface between the natural system and model system where the solution of the problem of anticipation was expected. Rosen looked at the coding through the interface between the abstract model and its real source as a creative act, stating that the coding did not belong to the system and its model.

The mathematical structure he and Rashevsky used was category theory (Mac Lane (1971), Rosen, 1985-2). Category is a generalized monoid on a set of *objects*. The objects of category theory can be various mathematical functions and structures. Monoid is a category with one object and semigroup is monoid without identity. What unites all these compositional structures is associativity and the absence of the axiom of inverse, which alone **does not mandate irreversibility**, however.

Rosen expected a complex system to be unpredictable, surprising, faulty up to a point, and exhibiting novelties. He, however, showed no affinity for thermodynamics of Ilya Prigogine who concentrated exactly on the surprise in the behavior of natural systems, but widely used *realistic* physical and chemical models.

Both Rashevsky and Rosen, following Bertalanffy, understood theory as a mathematical structure with irrelevant semantics, self-contained and self-explanatory, which, probably, worked to fence off the curiosity of empiric scientists.

A. C. Ehresmann (mathematician) **and J.P. Vanbremeersch** (physician; 1987, 1991, 1999), developing Rosen's ideas, came to the conclusion that natural complex systems "cannot be studied using observables defined on a fixed space of phases and following uniform laws" (1999). In their theory based on category theory, the set of elements in the *universe* is updated from state to state. It does it by "absorbing" some external elements and "suppressing" others. The efficient cause of an operation is a module called coregulator (CR), which seems to be an analogue of the enzyme in Gerhard Mack's universal dynamics, see below.

It is possible to formally remain under the axiom of closure because the system consist of a hierarchy of levels of complexity. The trade-off is the high complexity of the theory itself that has to trace all changes, ultimately, in terms of atoms. The theory, therefore, is, actually a representation of total available knowledge. The theory requires an *internal memory*. This requirement, in my opinion, is partly satisfied only in the case of the mind. Neither biosphere nor society do not have (yet) a centralized memory. As far as human history is concerned, only historians—but not the agents in AI sense—are in collective possession of such memory.

To compare with Pattern Theory (Section 6), pattern, according to Ehresmann, A.C. and Vanbremeersch J.P. (1999) is:

> A *pattern* in a category K is the data of a graph I and a homomorphism P from I to K.

Graph I here is the graph of arrows, i.e., associative morphisms interpreted as causal relations.

The statistical study of wars and conflict risk estimate by **Lewis F. Richardson** (1960-1993), whose studies of shorelines is said to inspire Benoit Mandelbrot's fractals, is closer in its scope to the early works of Nicolas Rashevsky.

Among Richardson's predecessors in quantitative history was **Pitirim Sorokin** (1937), a remarkable American sociologist and author of the multi-volume *Social and Cultural Dynamics*, whose life path once crossed with that of Lenin. The audacious step of Sorokin was to quantify a repeating historical *pattern* (such as war, revolt, etc.) not only in terms of *extensive* frequency but also *intensive* scale. This alone creates preconditions for hard science.

Richardson, whose main works were published posthumously, left a fruitful and lasting heritage, named after him research institutions, his works collected and re-published. His effort on collecting databases is continued on a large scale by **Peter Brecke** (Long, 2003) who is in charge of the project pursuing taxonomy of violent conflicts and identification of warning signs of conflicts **by methods of pattern recognition** at Sam Nunn School of International Affairs, Georgia Institute of Technology. This project might finally abate the historically well founded pessimism about the ability of humankind to draw lessons from its history. Clio has now a balance sheet. Notes on the methods of quantification of security are on the Web (Brecke, 2002). About the entire project, see Brecke (WWW).

There is an impressive literature on *Nonlinearity, Complexity, Combat & War* (War, WWW).

Among Richardson's results was the conclusion that military conflicts follow Poisson distribution. To generalize, historical events are rare and too many of them at the same time are improbable. This resonates well with the chemical paradigm where one or two, rarely three molecules are involved into a "chemical event." Both history and chemistry are local.

History, philosophy, and art—the realm of semantics—have been among life long interests of **Ilya Prigogine**, a Renaissance man and the founder of non-equilibrium thermodynamics and, arguably, the entire modern science of complexity. He repeatedly approached the problem of human history in view of the behavior of the complex non-equilibrium systems he was the first to study hands-on (Nicolis and Prigogine, 1989, Prigogine and Stengers, 1984). He has been especially interested in the problem of time and its irreversibility in history, in contrasts with classical physics and contrary to the Einstein's dictum "For us believing physicists, the distinction between past, present, and future is only an illusion, however persistent." (Einstein-Besso, 1972).

Prigogine's view of history has two major components:

1. Prigogine united many natural systems, which develop complexity and order, under the notion of dissipative structures. The eddies in a fast flowing liquid is an example. If there is a constant supply of work and its dissipation in the form of heat, the flow of energy keeps the system far from equilibrium and maintains order. A dissipative system can switch to different modes and manifest a coherence and order impossible at equilibrium. He suggested that life and society may belong to this class of systems.

2. Prigogine suggested that the irreversibility of historical time is a consequence of the series of bifurcations and symmetry breaking in the evolution of dissipative structures. According to Prigogine, it is the sequence of random choices that makes history irreversible. In terms of dynamical systems, it means that the phase trajectories of a dissipative system are diverging: such system does not possess Lyapunov stability. Prigogine compared its behavior with kneading the dough (baker's transformation). Slightly modifying his example, two raisins put in the dough side by side, will most probably diverge during the kneading. Note that the concept of stability of dynamic systems does not even involve thermodynamics with its energy, entropy, and temperature that need to be defined on social systems.

Prigogine has not applied his theory to history in any systematic way, yet some far-reaching conclusions follow. The dissipative system stays far from equilibrium **while the dissipation of energy lasts**. When it comes to an end or the system itself breaks down, it returns to an equilibrium or stays very close to it. Therefore, an LMS system, for example, an industrial civilization, cannot maintain its structure if either the supply of energy comes to an end or, which is sometimes overlooked, the dissipation becomes hindered because of the rising ambient temperature. The only alternative is a deep restructuring of the system. Dissipative system, as any thermal machine, is positioned between supply of work (order) and disposal of heat (chaos).

All such reasoning, however, as any doomsday scenario, is of little immediate concern for most people because the internal time of non-equilibrium thermodynamics is not in any way connected to the calendar time. The present pattern of industrial civilization emerged after 1700, when mineral energy had become involved into the process of dissipation by technology. Nobody knows when the global baker will apply his hands next time, but thermodynamics warns us where to look for possible trouble. It suggests a gauge for vital signs of a civilization jammed between mineral fuel and atmospheric temperature.

The concept of relational theory, with direct references to Rosen and further back to Rashevsky and, in most recent publications, to Ehresmann and Vanbremeersch, but **without mentioning** Pattern Theory, was expanded to a *universal dynamics theory* by physicist **Gerhard Mack** (1995 - 2001) who is looking for a "a universal chemistry in which general objects and links substitute for atoms and their chemical bonds," starting with a "preaxiom: The human mind thinks about relations between things or agents." The inception of the project was influenced by philosophy and Wittgenstein in particular. The further development of this project is planned to take a form of computation.

Mack's universal dynamics is relevant for the topic of history for two reasons. First, it covers not just a system but also a *history* of a system, which is itself a system called *drama*. Second, it advances a very fundamental idea of locality (first implemented by Maxwell's dynamics instead of Newton's action at a distance) in the following axiom where *arrow* is a term for morphism in category theory:

> 4. locality: Some of the arrows are declared direct (or fundamental); they are called links. All arrows f can be made from links by composition and adjunction, $f = b_n \circ \ldots \circ b_1$; $(n > 0)$ where b_i are links or adjoints [reversed arrows] of links; the empty product $(n = 0)$ represents the identity. (Mack, 2001)

The elementary act of change in the system, therefore, happens only in the **topological 1-neighborhood** of an object, where a composite link is made fundamental or a fundamental link is made virtual. When the last fundamental link is gone, the object dies.

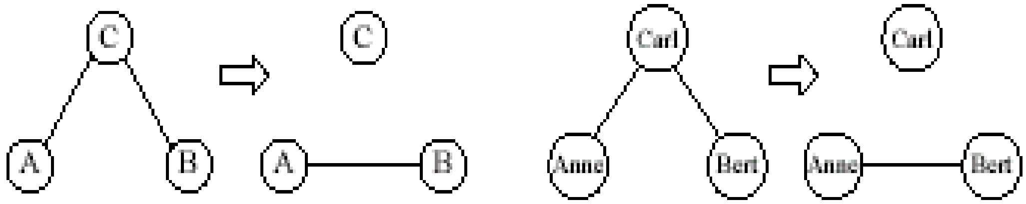


Figure 1: "Catalysis in chemistry and elsewhere. A catalyst C binds molecules A and B. First a substrate-enzyme complex is built, where A and B bind to C. Next the composite arrow from A to B becomes fundamental" (the Figure and caption are from Mack, 2001).

The atoms of a minimal change of the system are transformations of the graph of relations in the 1-neighborhood of a node: the edge can be either eliminated or established and the node can be duplicated. Mack distinguishes between the following four transformations, of which the first is the atomic one.

1. motion: "promoting" a composite arrow to the status of a direct one, i.e. link;
2. growth: by making a copy through a series of motions and duplications of nodes;
3. death: the only irreversible transformation, consisting of removing all the links;
4. cognition: approximately can be described as recognition; it establishes links between two non-atomic objects, which are, typically, isomorphic; cognition creates new arrows.

The closest to cognition phenomenon is enzyme-substrate interaction. The idea of the move is illustrated by Gerhard Mack with the example of generalized catalysis, Figure 1.

In universal dynamics the local transformation is performed by *enzyme*. It is not exactly what catalysis means in chemistry, but rather what *command* means in programming and *coregulator* in the system of Ehresmann and Vanbremeersch (1999): the efficient cause of a local change in the state of the system, i.e., a demon of a kind.

On the surface, the system under universal dynamics looks like a transformation of a connected graph that preserves its connectedness and occurs as a sequence of atomic changes one edge at a time. I do not see, however, how any new object can appear in the *drama* (which can be stochastic) and the notion of novelty does not surface in a mathematical structure under closure.

The question that arises when a compositional mathematical structures without a unique inverse is applied to a real system is: what if two causation arrows contradict each other? A similar question: what if two diverging arrows cause incompatible events? These questions do not apply to always "self-consistent" physical systems but are typical for LMS. I do not know the answers, although Ehresmann and Vanbremeersch (1999) allow in principle such contradiction of arrows.

For details one should better look into the sources. This project goes far beyond the initial relational concept of Rashevsky and Rosen and touches upon the deepest foundations of sciences. It has been evolving and some aspects lack details while others, such as theoretical physics, are beyond my competence.

A small set of minimal transformations can be found also in the relatively recent "Evolving Algebras" of **Yuri Gurevich** (1995) where the *arity* (number of arguments) of the transition function can be >1. Convergent or divergent incompatible outcomes would make computation impossible, but in political life, the right hand often does not know what the left hand is doing.

The idea of 16 *archetypal morphologies*, i.e., basic transformations of realistic structures, among them—adding or removing an edge or a node of a graph—goes back to **René Thom** (1973) who saw it as beginning, ending, fastening and cutting. Thom's ideas, unjustly trampled, (as Thom believed, because they did not deliver a quantitative prediction) , may be a key to the radical simplification of social complexity. Thom's *stirring* , which looks as a bell-shaped curve, can be identified with the transition state (see Section 6). the concept of catastrophe itself is what happens between the initial and final states.

From the chemical perspective, a set of two operations is sufficient to build a structure: form and delete a bond. In Pattern Theory (see the corresponding section), a node (generator) can be added and deleted. The confusing abovementioned situations in both chemistry and Pattern Theory are *irregular* , but natural: irregularity is just a degree of regularity, like chaos is a degree of order an *vice versa*.

Artificial intelligence used to ignore individual history or *story* (Schank and Abelson, 1995) of a robot or computer. The role and possible use of story in complex systems has been insightfully analyzed by **Chrystopher Nehaniv** and **Kerstin Dauterhahn** (Nehaniv 1998-1 and –2) and summarized as: "Being alive means having a lifetime, without a reset button." (Nehaniv and Dauterhahn, 1998-2) Nehaniv *at al* connected *story* (ontogenesis in biology) with the thermodynamic irreversibility of life and assigned to semigroups the label of "algebras of time." Nehaniv *at al* formalized the unique direction of ordering in semigroups and included "pools of reversibility" into the general semigroup structure that does not require irreversibility axiomatically. Nehaniv and Dauterhahn (1998-2) explicitly stated the problem of the limits that a closed mathematical structure imposes on the realism of representation but pointed to a "dynamically changing" what the system considers as its inputs and which are relevant as a possible direction of solving the problem.

Thermodynamics, which establishes a preferred direction of time, can hardly be classified as either syntax or semantics. It is external to timeless mathematical structures and finds no place there. The relation theories expurgate metrics together with semantics and find no place for thermodynamics. An intriguing case of thermodynamics on a formal but tainted with realism system is *reversible computation* **, Rolf Landauer** (1961**), Charles H. Bennett** (1973) . It arose from the problem of heat generation by the computers that have to erase all intermediate data and clear the registers after each computation and therefore, dissipate heat without creating information. See also Section 6 in Bremermann (1974) .

The information produced during the individual *history* of a finite state automaton, such as, for example, Turing machine, is discarded or, more generally, the mapping of a state back onto the previous state of a computation cycle is not bijective

The problem "how to save energy", therefore, resonates with "how to remember the lesson of history and survive the next election," in political language, as an alternative to starting the second campaign from scratch.

The reversible computing works on a **new** task by

1. **saving** all intermediate results,

2. generating the output,

3. retracing the computation back to the input, which is possible because of the saved history of the computation, while reversibly erasing only the history. While reversing the computation, a Turing machine that represents the finite automaton has to change the **read-shift-write** quintuple

$<AT \rightarrow T' \sigma A'>$

where A, A' refer to states, T, T' to the tape, and σ is the movement of the head

to the **write-shift-read** quintuple.

The overall result is the decrease in the dissipation of energy, which is still higher than in the direct computation alone but less than in the entire cycle of irreversible computation.

Motivated by the goal of bridging the gap between history and sociology. **Bertrand M. Roehner** (physicist) and **Tony Syme** (economist and sociologist) formulate their subject in *Pattern and Repertoire in History* as *analytical history* (Roehner, 2002). In their most innovative, wide in scope, and rich of facts work they follow the agenda of Charles Tilly (1981, 1984) who—very much in line with one of the stimuli that drive chemists and mathematicians—wanted to know whether the repertoire of elementary blocks in history really **existed**. Tilly, the student of Pitirim Sorokin, made clear that the binary relations between the blocks were of the primary interest for a historian-sociologist.

It may seem that, since history as a whole is a unique sequence of events, developing at specific times and locations, it can be only described as a whole but not studied by the scientific method. Natural sciences, since Aristotle and through Descartes, follow the method of analyzing a complex phenomenon and dividing it into simpler units. A closer look at history reveals that it is built of standard modules which repeat across time and space and can be compared and generalized in the form of *patterns*. The perception of wars, revolutions, and reforms as standard blocks of history is by no means new. The authors discover, however, that the smaller building blocks of the large events are also to some extent standard and enclosed in each other in fractal manner. Moreover, the smaller blocks are highly repetitive throughout time and geography.

Roehner and Syme note that real physical processes, like the warming of a cup of cold drink, are by no means simple. Only by splitting them into simpler problems physics managed to successfully explain them. They refer to simplicity as "a basic requirements of the human mind."

The main methodological idea here is to look for *patterns* or *regularities* in history at a microhistorical level, largely regardless of their macrohistorical environment. The obtained knowledge can be further used for explanations at the macrohistorical level.

To give an example, the apparently unique meetings of the French Estates-General are elements of a larger set that includes also the meetings of the parliaments of German and other European states of similar historical periods, while the French Revolution itself is an element of the set of European revolutions, successful, as well as abortive. The sale of Church property occurred also twice in England and once in Austria at completely different circumstances.

The comparative method, with its long jumps, lies beyond typical historiography, but the description of modules follows the historiographic tradition of a consistent narrative. To completely appreciate the wealth of material, the book must be read in its entirety. Many episodes of world history reviewed in it can hardly be found in general courses. Authors' analysis of difficulties and perspectives, as well as the task of a computerized Very Large Chronicle project is marked with the enthusiastic practicality typical for large scientific projects, although to recall the Manhattan Project would be politically incorrect.

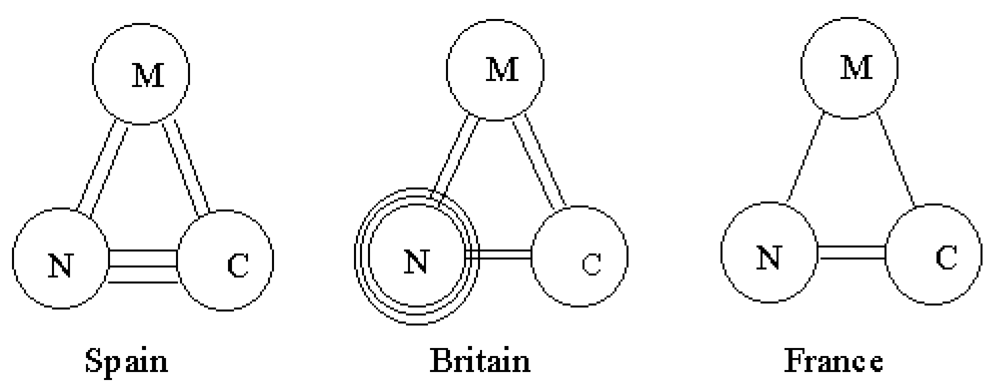


Figure 2. Monarchy, nobility, and clergy in three kingdoms, along Roehner, 2002, Fig. 2.1, p. 76. M = Monarchy, N = Nobility, C = Clergy

Any modern historical narrative can be illustrated by graphs, diagrams, and tables. In one case we can see a different type of a graph, see Figure 2 . It presents the relative strength of the relationship as the number of connecting lines. The concentric circles in the triangle of Britain reflect the fuzzy borders of nobility. Apparently, the weight of the influence can also be denoted by the area or appearance of the circle.

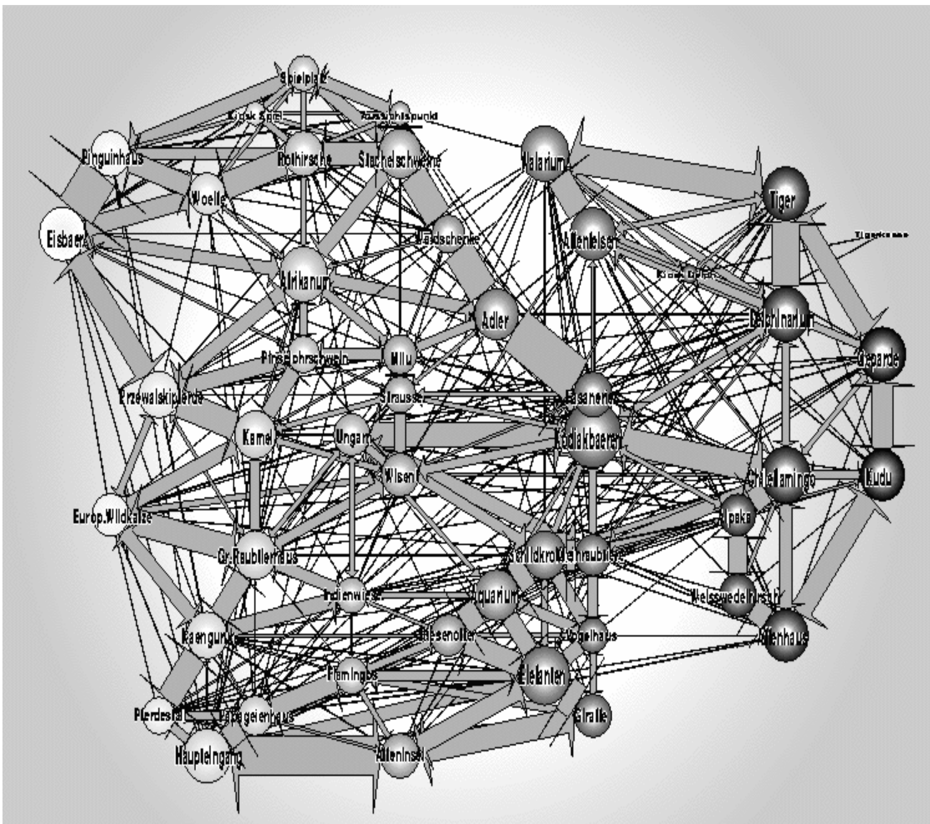


Figure 3. The structure of visits at Duisburg Zoo, **from Krempel 1. Source:** http://www.mpi-fg-koeln.mpg.de/~lk/netvis/zoo1.html

Analytical history naturally conjoins with non-equilibrium dissipative structures because, as Prigogine noted, complexity is what needs more words to describe it. Extending his reasoning, these necessary words should be taken from a dictionary different from that of simplicity. As "hard" scientists, Prigogine and his colleagues can always describe not just the wonder of organized behavior but its detailed mechanism for any dissipative structure, especially, of a chemical nature. Analytical history provides us with both vocabulary and structural elements from which the historical complexity is built and the authors are looking toward the syntax ”grammar” of it all.

To what extent a social structural chemistry can be quantified, the achievements of the fast developing trend of visualizing social networks (**Lothar Krempel**) show. This approach is strongly metrical. A number is associated with both nodes and edges of the graph, and all nodes are labeled with unambiguous semantics.

Two examples will do here.

Figure 3 presents the structure of visits at Duisburg Zoo (Germany). The size of the spheres corresponds to the number of visits and the width of the connecting lines gives the traffic. For example, the largest sphere in the bottom left hand corner is Main Entrance, from which one can go NW to Horses and from them to Kangaroos or NE to Parrots.

Obviously, we deal here with a very general representation. This picture can be *transformed* into an endless variety of configurations filling up a topological space with Hamming metrics. The neighborhood of a configuration includes all configurations with a single changed bond. This closed structure can be opened by expanding the neighborhood to **new** nodes, while their elimination would ensue after removing all links. The Duisburg Zoo must have had its own history in which sections were added, removed, and added again. This history would also reflect changes in traffic and number of visits. If normalized, the numerical values can be expressed as probabilities.

The second illustration, Figure 4, shows two snapshots of the dynamics of automobile trade 1980 and 1994. These configurations are already very close to those of Roehner and Syme.

A stunning variety of visualizations can be found in: Dodge (2001).

To conclude the review, the entire area of structural representation in humanities owes a lot to Claude Lévi-Strauss and structuralism (Lane, 1970). It seems that the initial wave of structuralism could not sustain itself in the cannibalistically competitive atmosphere of humanities with its generation wars and post-ism waves, but he is unquestionably among the founding fathers of the future consensus.

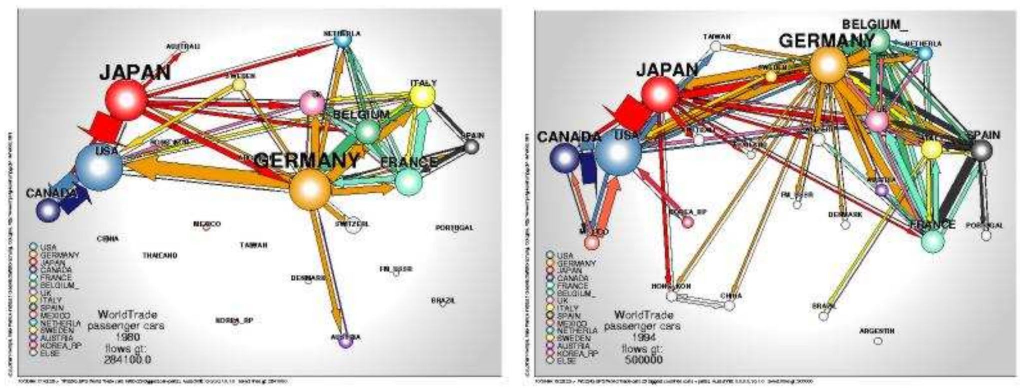


Figure 4. Dynamics of world automobile trade in 1980 (left) and 1994 (right). From Krempel 1. Source http://www.mpi-fg-koeln.mpg.de/~lk/netvis/global/autochap3.html

In this review I have left out a large area of mathematical sociology, already sufficiently naturalized. For the entrance to the area, see ASA (WWW). The volume of works in 2-neighborhood (and farther away) of “hard” history is enormous.

4. History and thermodynamics

Neither history, nor thermodynamics, nor, especially, mathematics belong to my professional area. The following is just an informal look from aside, aiming at some distinctive properties of Pattern Theory.

Reversible computing, although turned impractical, opened an entire and still unexplored area of using history for applied problems. Bennett’s paper (1973) is interesting also from another angle. It ends with a comparison of computation with mechanisms of molecular biology such as the template synthesis of messenger RNA and its reversible degradation. Bennett notes that although this process is also somewhat wasteful, it is much more efficient than irreversible computation. Bennett's description of the role and thermodynamic mechanism of catalysis stands alone with its high chemical accuracy.

What is known about enzymatic action fits well the framework of organic chemistry. Enzymes are proteins folded into a particular shape, which presents a real mystery. They easily solve the problem of their own folding into whimsical but reproducible shapes, which for humans is one of the hardest computational problems requiring brute force. This representational gap—it should not be hard for us what is easy for nature—may mean that it is not the final shape that is coded in their primary structure, but **the reversible history of folding**.

To illustrate the relation between history and thermodynamics, let us invoke again Maxwell's Demon, whose behavior can be mapped on the behavior of a human being. A biologist's argument against Demon can be as follows. Certainly, Demon can perform his job while he is alive. He would need energy, however, to process the visual information, send the efferent signals to his hands, and work against the door spring or, if there is no spring, against the inertia of the door. Therefore, the demon is possible, *modulo* his energy supply. He will have a history with a beginning and the end. This argument—only half-unserious—implies that in order to have a history, a system needs to dissipate energy, whether it consumes it or not. This argument does not even mention entropy.

In a mathematical system, the act of combination of some elements of a set into a subset, as well as any operation and mapping, does not require any energy, which is beyond the concept of mathematical structure. But why? Nothing in the nature of mathematics forbids us to assign metrics to the operation of combination. In reality—physical, chemical, biological, and social—two objects combine because either they have attraction to each other (i.e., the combined state has a lower energy than a separated one) or there is a source of energy to keep them together in spite of their mutual repulsion, or there is a constraint on separation.

Thermodynamics is absent from the syntactic representations of complex systems. Neither does thermodynamics belong to semantics. Thermodynamics follows from the idea that to each binary mathematical operation on the *real* system corresponds a change of some value (energy, free energy, attraction, stress, strain, etc.), which arranges such operations in at least a partially ordered set.

Coming back to the problems with the mathematical structures under closure, noted by Nehaniv and Dauterhahn (1998-2), the situation in a mathematical system is the same as with Prigogine's system where the phase trajectories not only diverge but also converge. The transition functions in a representation of the system **may not** have a single inverse and they form a semigroup, non-restrictive to reversibility. I tend to believe that an open mathematical structure for real systems should include "Prigogine axiom" which—scratching the ear of a mathematician—would sound something like:

> There is a distance $d = \Pi$ between the elements of a semigroup expansion (product of all partial histories, i.e., total history without repetitions) such as if $d > \Pi$, the expansion has no inverse.

Irreversibility in LMS does not follow from thermodynamics alone. It follows from an open character of the system in which information is erased and lost. For such a system to exist without essential loss of complexity, an acquisition of new information is necessary.

It is easy to formalize loss, but how to formalize novelty? Transformation is function *f* on set X:

$$f\colon X \rightarrow X$$

This leaves no place for either expansion or contraction of set *X*. Transformation f_o on an open set would mean:

$$f_o : X_t \rightarrow \cup\ (X_{t+1}\ ,\ \Delta X_{t+1}\ )\ ,$$

where ΔX_{t+1} is a change of the universe of the mathematical system.

To abandon the axiom of closure, however, would mean a profanation of the entire mathematics. Bourbaki, probably, realizing that, ended his *Set Theory* (Bourbaki, 1968) with a strange construct: the set which is the mathematical image of emergence: a combination of elements of the basic set becomes a **new** element in an expanding and partially ordered set: the scale of sets.

In a very general sense, there is an obvious but rather metaphoric relation between gauge theories in physics (Moriyasu, 1983) and any theory invariant **regarding the change of the base set**. It seems that Pattern Theory is the only one developed theory of this kind. Probably, only a very abstract mathematical analysis can answer the question whether the world from physics to poetry is essentially uniform. From the point of view of PT, this is certainly so. Intuitively, it seems that Robert Rosen was right and simple systems are rare exceptions in the curved evolving universe. It is interesting to see Gerhard Mack's progress.

The evolving scale of sets with forgetting and loss of elements, balanced with an inherent expansion is, in my opinion, a basis for an open associative mathematical structure that does not contradict the axiom of closure because the basic set, which for real LMS systems is the alphabet of a language or the elementary percepts (cells of the retina, tactile receptors, etc.), remains constant at least in the medium run. The function of the language is to create a sufficiently large combinatorial space for a limited size of the combination. Taking an average length of a word as 5 letters and alphabet of 25 letters, the abstract word space contains 5^{25} words. Only a tiny part of this space is occupied by existing words, as a tiny part of the display pixel space is occupied by meaningful images. This property of an extremely sparse use of combinatorial space is an intrinsic property of LMS.

Instead of describing LMS in thermodynamic terms as systems far from equilibrium, we can describe them as systems that always occupy only a very small part of all available combinatorial space. Complex systems, therefore, are **simpler** than they could theoretically be.

Although any concept and even a word of the vocabulary has its own history, this history is usually forgotten. For example, the words war, and chemical bond have completely lost their histories, while the word rose is still ties its meaning to some sensory perceptions. In this way, a knowledge representation becomes an open system.

5. Pattern Theory

A mathematical theory that can be recognized by a chemist as meta-chemistry was first introduced in the 1960s, around the same time as "categorical" theory. Pattern Theory of **Ulf**

Grenander (1976-2003) assigns metrics and structure to the naked operation of composition. It incorporates not only atomism and Klein's geometry (transformational invariance) but also opens the door to thermodynamics. Moreover, it opens to openness itself. To an eye of a chemist, it does it by realistically implementing atomism and the principle of locality.

Constructs of atom-like elements connected in a certain order are the original subject of chemistry, not borrowed from physics. Both chemistry and sociology, as well as a large number of other disciplines, regardless of whether they are of human or inanimate origin, describe objects of various nature and complexity in terms of their structural elements: building blocks and bonds between them. Pattern Theory is the mathematics—and universal chemistry—of such objects.

The papers and books on relational LMS are conspicuously devoid of realistic examples. On the contrary, the first impression one might get from a book on Pattern Theory (Grenander , 1995) is the cornucopia of illustrations as different as skeletons and languages, fairy tales and stomach shapes, mitohondrias and clustering of human settlements, with a potato on the cover and two color plates of battlefield paintings inside.

Pattern Theory, when first introduced, was based on four principles:

(1) **atomism** of building blocks (generators) possessing a selective ability to (2) **combine** with each other according to rules of combination, which leads to regular configurations that are (3) **observable** as images. The fourth principle is (4) **realism**:

> A theory of patterns that would not take into account how actual patterns behave would be severely limited in its applications. We must, therefore, assure that the theory is realistic so that it can deal with real patterns (Grenander, 1976, p.3).

This is the kind of realism an empirical scientist would expect from a theory. Let us note that by building blocks Ulf Grenander understood also "abstract symbols, sets, relations, or functions," which puts it at least at the same level of generality as category theory.

NOTE ON ATOMISM. Atomism is traditionally attributed to Democritus and Lucretius. Regarding the philosophical roots of modern mathematical atomism, it seems unfair to omit Leibniz with his monadology: "

"3. These monads are the true atoms of nature, and, in fact, the elements of things.

……………

8. Whatever is in a composite can come into it only through its simple elements and the monads, if they were without qualities (since they do not differ at all in quantity) would be indistinguishable one from another (Leibniz, 1973).

Remarkably, as if anticipating the universal dynamics, Leibniz, an antipode of Newton, explained the interaction at distance by the composition of local interactions.

The atomistic principles, based on the variety of qualities of the atoms, underline Pattern Theory, in which (informally):

1. Primary atomic elements (generators) have labeled bonds (bond structure), similarly to atoms in chemistry.

2. Generators with similarly labeled bonds form bond couples, thus combining into configurations, similarly to molecules in chemistry. In Grenander's language, the generators **interact**.

3. Both generators and their binary composits (bond couples between them) can be attributed a rich variety of properties, including equivalence classes and numerical values, such as, for example, the acceptor function that defines the selective affinity of generators toward each other. Pattern system can be fine tuned. Analysis and synthesis of patterns relies on heuristics as much as on formalism.

4. In a closed system, probabilities of configurations are determined by probabilities of generators and bond couples, similarly to chemical equilibrium and chemical additivity.

5. Observable images are introduced as equivalence classes in configuration space, similarly to classes and conformations in chemistry;

6. Regularity R is defined on a set of configurations as quartet $R = (G, S, \rho, \Sigma)$, where

G is generator space, S similarity transformation defining equivalence classes in generator, configuration, and image spaces, ,ρ is bond value relation for coupling bonds, and Σ is **type** of connector graph, which, in essence, is also defined trough local properties.

Regularity in chemistry is called *stability* : the property of a chemical structure to be isolated and stored for significant time. In both PT and chemistry irregularity means higher energy as compare with the regular structures.

7. Template is a representative member of an equivalence class of regular configurations: other regular configurations can be obtained from it by group of similarity transformations. Thus, under a particular (rather loose) regularity *R*, an indefinite number of sociological and economical structures can be obtained by transforming Figure 3 *modulo R* . Given the Periodic Table and regularity of chemistry, the entire chemistry can be built, bond by bond, from any compound and elements, which is, by the way, not only is feasible in the lab but was the actual origin of biochemistry on the Earth. The problem becomes much more complicated in sensory, and especially, visual images.

The realistic character of Pattern Theory makes it highly recognizable as a good "meta-chemistry" for a chemist. I have already discussed the chemical realism of PT (Tarnopolsky, 2003). Moreover, although PT is *algebraic* in the treatment of groups of transformations that identify images as equivalence classes of configurations, the set of generators **may be open**. This idea was expressed by Ulf Grenander in his first large pattern system MIND (Grenanser, 2003, p.7):

> The set of all generators available to a particular mind will be denoted by G ,the *generator space* . As time goes on G may change: new primitives may be acquired, others forgotten, but for the moment we shall treat the generator space as fixed.

This seems like a radical idea for a mathematical system, and, for that matter, even for the systems theory. MIND contains other radical ideas, among them, that only a small part of the generator space (content) is involved into the ongoing transformation of the mind and, moreover, the content is drifting in a regular (in the sense of PT) manner through the configuration space. It means, in my interpretation, that the mind, unlike the computer (and unlike a robot in AI), has no random access memory: the **most probably accessible memory** is the 1-neighborhood of the generators in the content. The depth of memory can also be accessed, but at lower probability, as a "sudden revelation" or "creative act." The next state of the content is formed from the generators connected to the previous one. This prevents the degeneration of configurations to negligible probabilities. The mind is a device for self-discipline.

I would interpret Grenander's MIND as not a point but a **cloud of probability** moving through the phase space of the system. In the present version of MIND the model is, essentially, ergodic, which an evolving LMS system is not: the configuration space changes through inputs, forgetting, and the steps of the Bourbaki's scale. Of course, ergodicity makes no sense whatsoever for very large combinatorial spaces with times of search exceeding the age of the universe. For example, the pixels of a digital image generate over $2^{1000000}$ combinations.

> NOTE: To make a pattern-theoretical equivalent of an open system, one only needs to establish a competition for a limited resource between configurations. Loss of information is an equivalent of dissipation, while acquisition of information can be arranged through the Boolean function of *novelty* (Tarnopolsky, 2003).

The model of MIND reproduces the important property of LMS: the change is local. It means that changes in all sufficiently large system occur in limited areas. To put it differently, significant events in LMS, like earthquakes, are expected to follow Poisson distribution and a large number of simultaneous changes is improbable. What was confirmed by Lewis F. Richardson for wars may be true also for Washington scandals.

Another radical idea of Pattern Theory seems to be directly opposite to the exclusively relational approach. Both generators and relations between them (bond couples) can be associated with labels and numbers, for example, bond values, probabilities, or energies, as well as global **temperature**, which makes Pattern Theory a kind of intrinsically thermodynamical mathematical system, deeply analogous to chemistry. Given the regularity, it invisibly "calculates" its own probability distribution over the configuration space in the same manner the chemical system "calculates" its equilibrium or proteins "calculate" their folding. Monte Carlo methods are used to compute this equilibrium, which otherwise would require as much computation as protein folding.

The best way to Pattern Theory, with its unique combination of abstraction and realism, is to turn to the original sources. My immediate goal is to see what it can do for history.

The pathway from Pattern Theory to history goes through chemistry, which has been a metaphor for LMS, starting from Leo Tolstoy who regarded big historical movements of people and ideas as fermentation.

6. Chemical kinetics: the short run

I discussed the relation between PT and chemistry elsewhere (Tarnopolsky, 2003). Here I will focus on the concept of transition state from the meta-chemical perspective, i.e., in terms of generators and configurations of PT, which significantly idealizes the real chemical situation.

The concept of transition state (Eyring and Polanyi,1931) is a very general concept of dynamics.

> Transition state theory (TST), introduced by Eyring and Polanyi … in 1931 as an early attempt to determine absolute reaction rates, is too often considered the domain of the chemist or chemical physicist. However, the transition state (TS) is actually a general property of dynamical systems which involve an evolution from "reactants" to "products." Such processes include, but are by no means limited to, the ionization of atoms, the dissociation or re-action of molecules, and even the escape of an asteroid from its orbit (Jaffe et al, 2000).

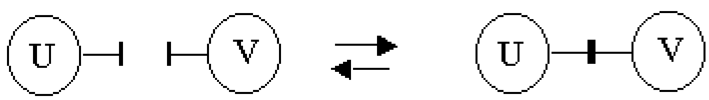
The transition state theory is the central explanatory paradigm of chemical kinetics. It assumes that between configurations A and B on the same generator space the position of equilibrium is defined by:

$$[A]/[B] = K_e \; ; [A] + [B] = C;$$
$$\log K_e = - (G_B – G_A) / kT \;\; ,$$

where [A] and [B] are probabilities (or concentrations, in chemical terms), C = 1 for probabilities, C is total concentration in chemistry, K_e is equilibrium constant, and G_B and G_A are energies of the two configurations, additive over generators and bonds. They are Gibbs energies $\Delta G = \Delta Q – T\Delta S$ in chemistry, where Q is thermal energy, T is temperature, and S is entropy.

In such a simple system, energy and probability are just two scales—linear and logarithmic—to measure the same parameter of distribution. In real chemical systems, absolute values of energy are rarely observable or calculable and in real complex stochastic systems, the event space is never complete. For all practical reasons, chemists are satisfied with their differences ΔG, often even with the sign of ΔG.

If configurations A and B are formed from only two generators U and V, the transformation consists of bonding and unbonding of the generators:



A B

No other configurations are possible and nothing else can happen.

On rich generator spaces, combinatorial configuration spaces, under the same regularity, can be indefinitely large. In this case configurations can be in equilibrium with indefinite number of other configurations (Figure 5).

The chemist tacitly accepts this, but on one condition: the time of equilibration is infinitely large. If the time of equilibration were short, explosive chemical complexity would be unmanageable.

In fact, only a small fraction of all possible transformations runs with any measurable speed. The chemists possesses a large (but not exorbitantly large) collection of heuristics, which allow for limiting the alternatives, ideally, to one, two, or otherwise accept the uncertainty, as in radiation damage of biopolymers. The chemist simply knows that some bonds, actually, most of them, will not change in a spontaneous movement toward equilibrium. Only the fastest transformations can be taken to account.

Suppose, we have two concurrent transformations:

$A + B \rightarrow C + D$

$A + B \rightarrow F + G$

Since the rate of transformation $AB \rightarrow CD$ is proportional to the product of the probabilities (concentrations) of the configurations on the left, the fastest transformation will quickly exhaust A and B and further slow down $AB \rightarrow FG$. The two transformations compete for a limited resource. Nevertheless, if both are reversible, the final result will be equilibrium. By skillfully using what is called thermodynamic (equilibrium) and kinetic (transformation speed) control, and, especially, catalysis, the chemist rides the professional bicycle that falls down when it stops. The living cell is no different: it must run its biochemical cycles in order to stay on the road.

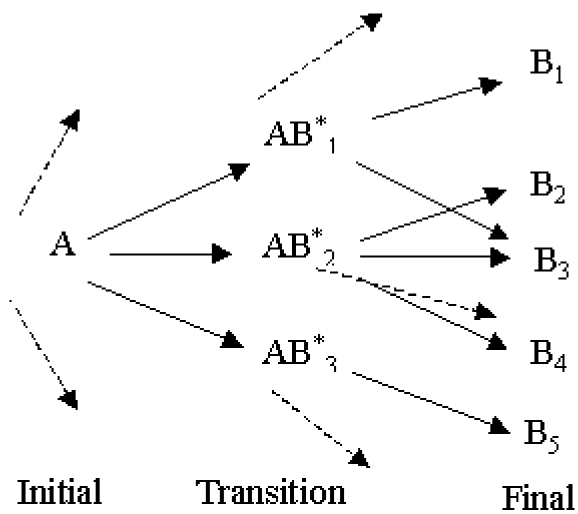


Figure 5 The tree of transformations in chemistry

The key to managing chemical complexity and staying away from combinatorial explosions is in the reaction rate, which for $AB \rightarrow CD$ is expressed as:

$d[C]/dt = d[D]/dt = K_r[A][B]$, where K_r is the rate constant.

To explain why rate constant vary and transformation takes some time, the chemist assumes that between A and B on one side of the transformation and C and D on the other side, all four being *regular*, there is an *irregular*—and therefore unstable, ephemeral, and of low probability/concentration—transition state $ABCD^*$, which is neither AB nor CD. It is much less probable and, therefore, of higher energy than AB and CD, so that AB and CD are in equilibrium with ABCD*. The transition state, therefore, is the bottleneck of *real* transformation.

It is easy to see, that this conjecture introduces time into equilibrium thermodynamics where it does not belong. Justifications for this sleight of hand can be found in quantum physics.

In Figure 6 transformation $A \rightarrow B$ ($G_A > G_B$) goes through transition state AB^*. The reverse transformation $B \rightarrow A$ brings the system, sooner or later, to the equilibrium. The appearance of the curve is entirely fictional because it is not observable. Moreover, in most cases, AB^* is not observable either: the entire curve is passed very fast. The bottleneck is caused by the unfavorable position of the equilibrium between A and AB^*.

In social systems, it is the very rarity of a "big" transformation, as the rarity of a formative event in Freudian psychology, that makes it irregular.

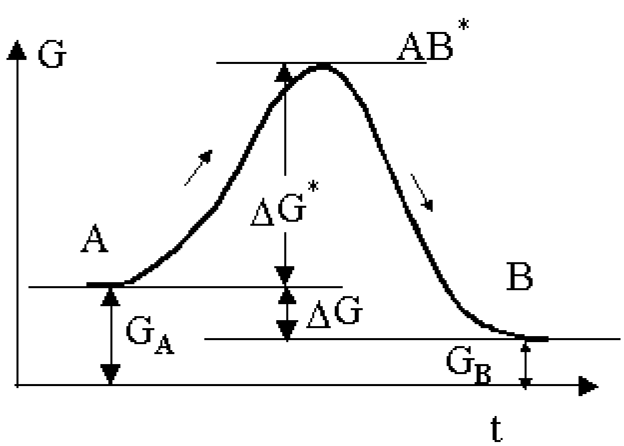


Figure 6 Transition state AB^* between stable states A and B

The fact that the transition state is typically not observable follows from its instability; the instability (short life span, low concentration) follows from its high—as compared with stable initial and final states—energy; the high energy follows from its irregularity; irregularity is most often the result of a loose uncoupled bond or another internal *stress*, for example, because of geometrical *deformation*. The italicized words are used in similar meanings in social psychology (stress) and PT (deformation), not to mention material engineering. What follows from this causal chain is the notion of regularity and stability as the vehicle of analogy across long distances in knowledge representation.

The sequence of stages, including hypothetical or sometimes observable (for example, by color or magnetic properties) transition states is called *mechanism*.

Example: transformation $A—B + C—D \rightarrow A—C + B—D$ can go through two mechanisms:

1.

$A—B \rightarrow A^* + B^*$;

$A^* + C—D \rightarrow$ **[A·····C·····D]***$\rightarrow A—C + D^*$;

$D^* + B^* \rightarrow D—B$

2. $A—B \rightarrow A^* + B^*$;

$C—D \rightarrow C^* + D^*$; $A^* + C^* \rightarrow A—C$;

$D^* + B^* \rightarrow D—B$

In the above mechanisms, all configurations with asterisk, i.e., with missing or extra bonds, are unstable. In $[A\cdots\cdots C\cdots\cdots D]^*$ the unusual (irregular) dotted line symbolizes irregular bonds. It can be compared with a love triangle, while A^*, B^*, etc., represent the singles eager to find a partner. Let us have in mind, however, that what is irregular in chemistry is perfectly regular in quantum chemistry.

To summarize, in pattern kinetics, irregularity is the key to the probability of a certain direction of change.

7. Equilibrium, kinetics, and catalysis in history

As applied to history, the **hypothesis** is: if two or more alternatives are available, as in Figure 5, what can go through the least irregular transition state, i.e., with the lowest energy, relatively to the initial state, has the highest chances to happen indeed. Of course, the word “probability” is used here metaphorically, as we use the words “probability of rain” talking about weather. It would be more appropriate to use the vague “chances.”

In the long run, however, the outcome is driven by the difference in energy of the initial and the final state. Thus, in the full of pattern trajectories traceable through 2500 year distance in time “*The Peloponnesian War*” Donald Kagan (2003) comes to the conclusion: “In a war of attrition the side that does the most damage must ultimately win.” (p. 75). He obliquely suggests that if the Athens were more offensive against Sparta in the very beginning of the war, instead of taking a defensive position, the outcome could be more favorable for it. In fact, a period of a more active military policy in the middle of the war brought the Athens significant success, lost later.

Similarly, Paul Kennedy believes that the outcome of WWII was predetermined by the economic and numerical superiority of the anti-Nazi allies who, I would rephrase, could inflict more damage. In the short run, the Nazis seemed unstoppable. But the history of the twentieth century is full of conflicts where the superior power could not win.

The final state of a historical conflict, therefore, can be evaluated more or less objectively. However, quoting Kagan, “But reason rarely predominates when states and their people have gone to war, and objective calculations of comparative resources are rarely enough to predict the course of an extended conflict.” (p. 63). People are driven by “fear, honor, and interest.” Besides, as Kagan noted, true democracy can be a big “inconvenience” for a country at war (p.87). In the first American war of the twenty-first century, the unwillingness of the democratic societies to inflict damage on anybody, including themselves, was unprecedented.

Turning to the most recent events, the Second Gulf War of 2003 against Saddam Hussein had an extremely low transition barrier: the well oiled military machine could be started by a single order, and Iraq had no air defense. The transition to the post-war situation was swift and smooth. The final state, however, had opened a long-run process, with its own hills and valleys of energy landscape, where completely different forces began to act in the war of attrition. Only a historian, however, can offer the postmortem of the final state. While history is in the making, the final state is in the imagination of the leaders and the public for whom the transition state is of a more imminent priority.

Strictly speaking, **historical process has no *a priori* known final state**, which is in full compliance with the view of Prigogine. Extrapolating this principle on history of science itself, however, we **may hope** (exactly because the outcome is not known in advance) to have some better understanding of patterns of history, including the transition and final states, after developing some theoretical foundations and processing the Very Large Chronicle by the apparatus of analytical history. History of science is full of realizations of impossibilities. The way to such understanding lies through analytical history, which was simulated by an ontological question. Similarly, we may ask whether a repertoire of initial, transition, and final states exists. The repertoire may turn out much simpler that we could expect because of the locality of change: historical patterns involve small configurations, as Grenander's *content* involves a tiny fraction of the *envelope*.

Chemical transformations take time because, due to the Maxwell-Boltzman distribution of energies, only a small part of molecules have sufficient energy to cross the barrier of transition state (it is called *activation energy*). Anything that could lower the barrier of a particular transformation would enhance its rate. This is the essence of catalytic effect which can be quite dramatic. The enzyme catalase increases the reaction rate by over 100 times, which reduces the reaction time from years to seconds. This reaction goes to the very end because the removal of the forming oxygen makes equilibrium impossible: Peroxide → water + oxygen , or

$H_2O_2 \rightarrow H_2O + O_2$.

A common misconception about chemical catalysis is that it works as in Figure 1, very much like two hands of an assembly worker (demon), by taking two pieces, connecting them, and an disengaging itself from the assembly. The devil of this picture is in the reversibility of time. The catalyst will enhance both direct and reverse chemical transformation, or, in terms of a match-maker, both marriage and divorce. The mechanism of chemical catalysis is beyond the scope of this paper (a quick view of kinetics, catalysis, and transition state can be found on the Web). If chemistry moves toward equilibrium, while biochemistry does not, it because of the dissipative nature of life: the wheels of biochemical cycles spin in one direction, consuming free energy and ejecting heat. The same fully applies to civilization: micro-events could be reversible—as the prohibition law of 1920 was—but the overall is not—**while dissipation lasts**.

Therefore, we can call *catalyst* any factor that lowers the transition barrier and *inhibitor* any factor that rises the barrier. Thus, the military attack of USA on North Korea is currently strongly inhibited by the geography and resources of the Korean Peninsula, however attractive the final bliss of security looks in imagination. The catalyst is the fear of the consequences of a North Korean contribution to a nuclear attack on the USA. It is easy to imagine the high irregularity of the current transition state and the stress it imposes on the decision makers.

There is another powerful factor that can lower the transition barrier: high temperature, i.e., the intensity of chaos. In times of confusion after a shock of an attack, natural disaster, or mass revolt, the high abstract temperature indiscriminately decreases all differences between the energy levels and flattens out the energy landscape: anything seems possible and the head of Louis XIV falls into the basket.

8. Illustration: Darius against Scyths

Let us consider as an example a fragment of ancient—to avoid any contemporary political bias—history as told by Herodotus (1955).

The story of the Persian march on Scythes, around 500 BC, would make a great script for an action movie if there were lot of action or at least a love story. In fact, there was mostly inaction: Darius did not manage to engage into a battle with the Scyths. And yet the story is full of suspense. Herodotus gives a vivid account of the struggle of the minds of the opponents that brought the conflict exactly to the point where it started. The story is remarkable by the psychological coherence and explanatory details that show the causation of events. It is both a history and a literary story.

The pattern of the story—the expedition and its return without an accomplished goal—can be seen, probably, in the Vietnam War and some other conflicts of the last half of the twentieth century.

Herodotus gives the lack of allies and any land to protect as the reasons why the nomadic Scythes initially refused to fight. When some of them finally decided to give a battle, it was too late: Darius decided to go home. The reasons for his final decision are stated with high psychological accuracy.

When Darius, desperate of the void that meets him in Europe, challenges the Scyths, they respond with a cryptic message allowing for a multitude of interpretations. The *uncertainty* of situation brings Darius under stress. He is inclined to see the message as the sign of surrender, but Gobryas, his general, sees it as a threat. Next, the equally cryptic behavior of Scyths who abandon the battlefield to chase a hare, makes the stress unbearable and shifts the unstable transition state of cognitive dissonance toward the unfavorable interpretation of the message. This finally prompts exhausted Darius to turn around and go home.

My purpose here is to trace the mechanism of the expedition as a sequence of stable configurations and transition states.

There are at least three outcomes in a war: victory, defeat, and tie. The latter can ensue as result of equal loss or a voluntary avoidance of the battle and return. In the initial simple configuration **Darius—attack—Scyths** we have no data to select the final outcome. We need to build up the complexity of the initial state to decide in which direction it could possibly move.

Thus, Darius is not just a person, but a king, his location is Asia. He possesses a great empire and has nothing to lose but a bit of his prestige. The Scyths have nothing to lose because they are not tied to the land. This makes the conflict a kind of a cold war.

At the beginning of the story Darius only contemplates the expedition. Herodotus supplies us with almost all available (but not always reliable) knowledge about Darius and the Persians, even such details as the thickness of their sculls as compared to those of the Egyptians. We can start, for example with the expansion in Figure 7

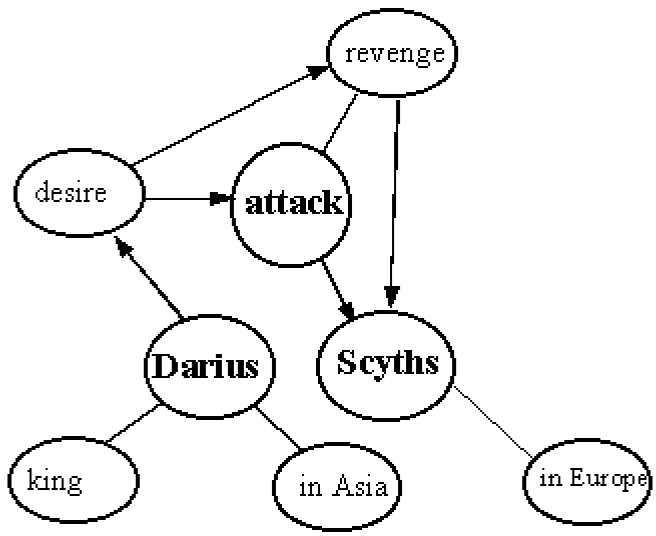


Figure 7. Darius and the Scyths: the seed of an initial configuration

By adding more details we can build an incomplete initial configuration as in Figure 8 . It consists of two sparsely connected parts around Darius and the Scyths. A complete one would include all we know about the events, protagonists, and their physical, historical, and geographical backgrounds. For example, a separate subconfiguration would represent Ionians who double-crossed the Scyths. We do not need this, however, because most of the factors are either irrelevant or remain unchanged.

None of the configurations, graphs, tables, and other representations has any claims for the truth. My purpose here, following the ideas of Ulf Grenander, is to show how this can be done in principle, so that a platform could exist for **debates** possibly leading to a relative consensus typical for natural sciences. As I believe, that type of consensus was in the ultimate vision of Pitirim Sorokin and Vilfredo Pareto in sociology. In is the same way of *naturalization* social psychology has managed to create a platform for discussion and consensus in the area of small groups with the theories of balance (Heider, 1958), cognitive dissonance (Festinger, 1962), etc., having a strong chemical flavor. Apparently, Benedict Spinoza was looking in the same direction in his *Ethics*.

In the absence of quantification, the measurements can be reduced to ordered and partially ordered sets and approximate scales, called "naïve" in Artificial Intelligence. As a curious example with historical flavor I would quote *Analects* of Confucius (WWW) where the Master builds a partially ordered set of ethical values:

> Tsze-kung said, 'What do you pronounce concerning the poor man who yet does not flatter, and the rich man who is not proud?' The Master replied, 'They will do; but they are not equal to him, who, though poor, is yet cheerful, and to him, who, though rich, loves the rules of propriety.' Book I, Chap. XV. 1.
>
> The Master said, 'They who know the truth are not equal to those who love it, and they who love it are not equal to those who delight in it. ' Book 6. Chap. XVIII.

In the same way, I measure the irregularity—otherwise, tension or stress—of the situation on the scale from one to four.

The **Table** lists some states of the story. The tension is marked by 1 to 4 red asterisks. The moments of irregularity are those uncertain and short-living transition states where, like in the chemical transition state $[A\cdots\cdots C\cdots\cdots D]^*$, the rules of regularity are violated. For example, when a decision **is being made** or the protagonists encounter a **puzzle** that does not fit any rational framework.

The focus of the tension, shown by a red circle, usually has two incompatible arrows converging on it (disputes) or two diverging (decisions) arrows of alternatives. Those are Prigogine's moments when the baker again folds and rolls the dough of the process. In the Antiquity, the baker was a god. Human decisions and actions, of course, may have zones of reversibility.

Figures 9 and 10 present some stressed intermediate states and the final tension-free state of the last decision and a dubious happy end of the safe return.

The intermittent stable and transition states are shown in Figure 11. The "mountain chain" or "roller-coaster" appearance is typical for stories, whether fictional or real. For comparison, Figure 12 portrays the roller-coaster of the French Revolution.

Table: The events and tensions of Darius' expedition

No.	State. **D**: Darius, **S**: Scyths	Tension / irregularity No.	Persians	Scyths
1	Darius wants to attack Scyths			
2	Artabanus tries to dissuade **D** in vain.	1	**	
3	**D** sets out			
4	**D** passes two rivers			
5	**D** Passes Ister. Dispute whether to leave the bridge	2	**	
6	Scythians confer with local people, looking for allies. **S** decide not to fight **D** in a battle.	3		**
7	**S** lure **D** eastward, over Tanais (Don).			
8	**S** direct **D** into non-allied people, to provoke them. Send the wagons north.			
9	**D** reaches Oarus (Volga), sees no **S**, and turns back.	4		**
10	**S** meet resistance of non-allied people.	5		**
11	**D** send a message to **S** (to Idanthyrsus).	6	***	
12	**S** send troops to Ionians at the bridge, others decide to fight.	7		***
13	Braying of the asses bothers **S**	8		**
14	**S** try to delay **D**'s departure and help him with food.			
15	a bird, a mouse, a frog, and five arrows	9	***	
16	**S** confer with Ionians, who agree.	10		***
17	Almost a battle, if not for the hare.	11	****	
18	Gobryas: leave the weak, move to Ister			
19	**S** pursue **D**, going to the Ister but the armies miss each other			
20	The Ionians decide on the fate of **D**	12	**	
21	Ionians give false promise to **S** and destroy only a part of the bridge			
22	**S** believes the Ionians fro the second time			
23	**D** finds the bridge broken	13	**	
24	An Egyptian with a loud voice saves **D**, calling, by his order, Histiaeus, who responds from the other side			

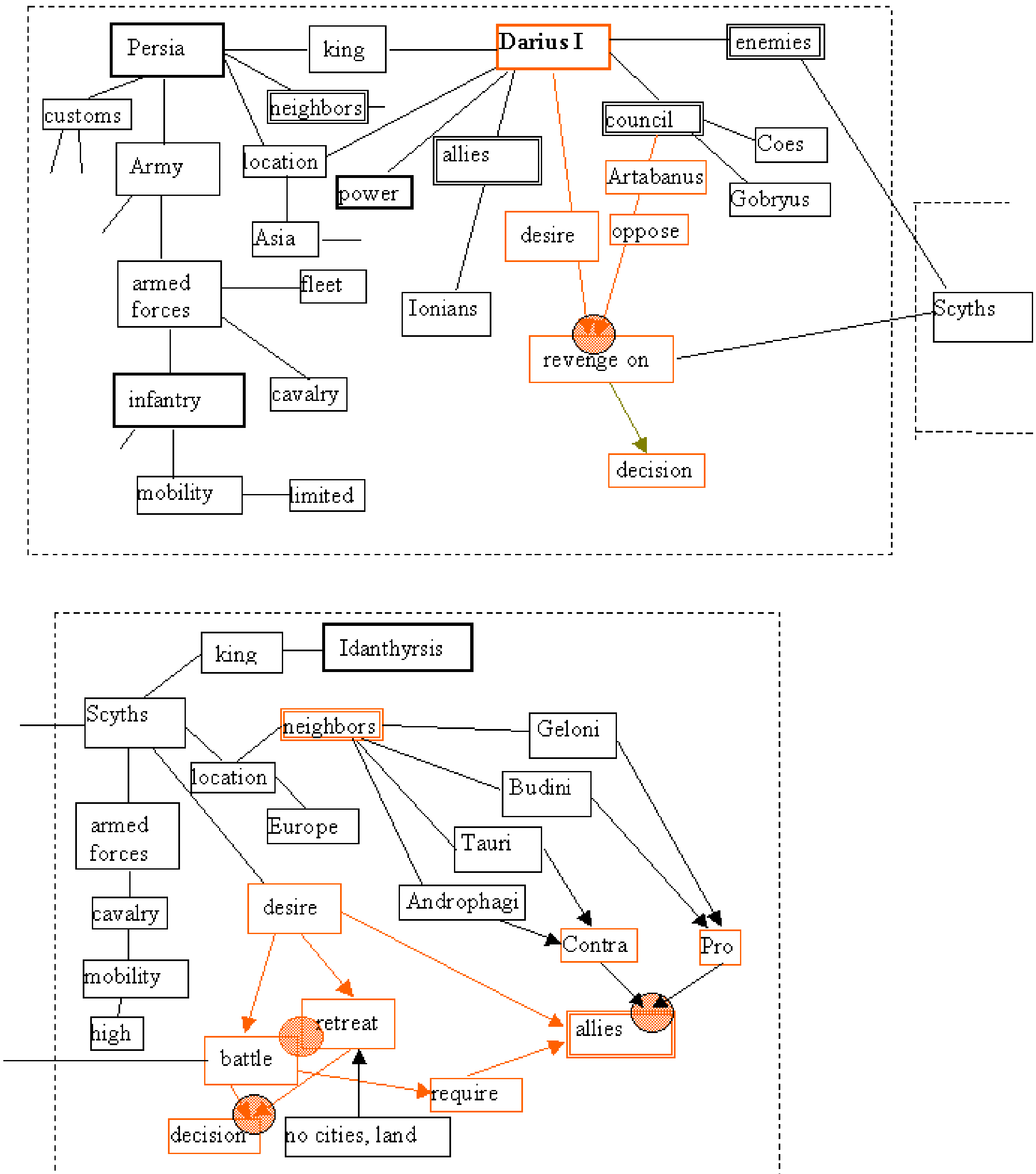


Figure 8. The initial irregularities of Darius and Scyths configurations (red).

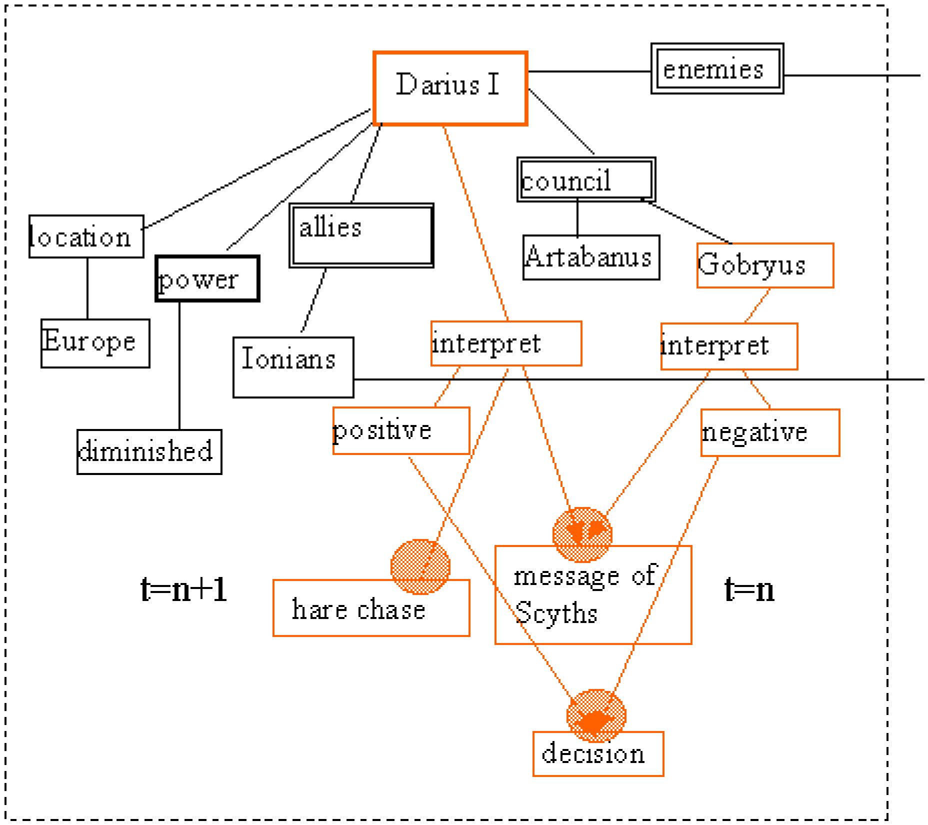


Figure 9. Intermediate irregularities of Darius configuration.

Figure 8 consists of two sparsely connected subconfigurations of Darius, State No. 2 in the Table, and the Scyths, State No.6.

Figure 9 presents States 15 and 17, separated in time, as marks t=n and t= n+1 show.

Figure 10 portrays the 20 and 21, i.e., the process of decision-making by Ionians. The final state is still slightly strained by contradictory interests of power and independence.

Figure 11 corresponds to the final state of Darius configuration.

Figures 12 and 13 illustrate the landscape of tension (i.e., relative energy) in a series of relatively stable and transitional configurations for the march of Darius and the French Revolution.

It must be emphasized, that the illustrations aim at the goal of building a platform of consensus, but not the consensus itself, which should be left to specialists.

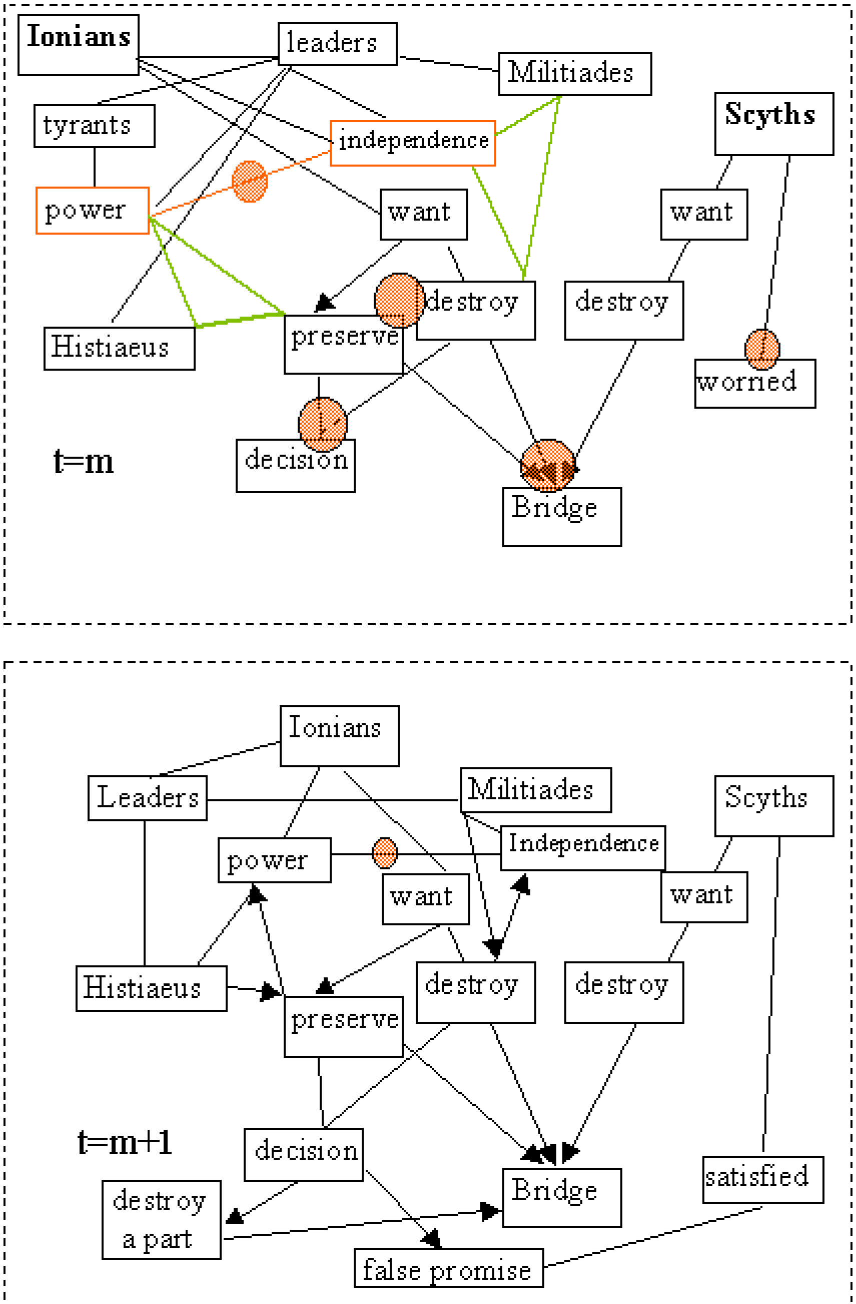


Figure 10. The transition (above) and final (below) states of Ionians.

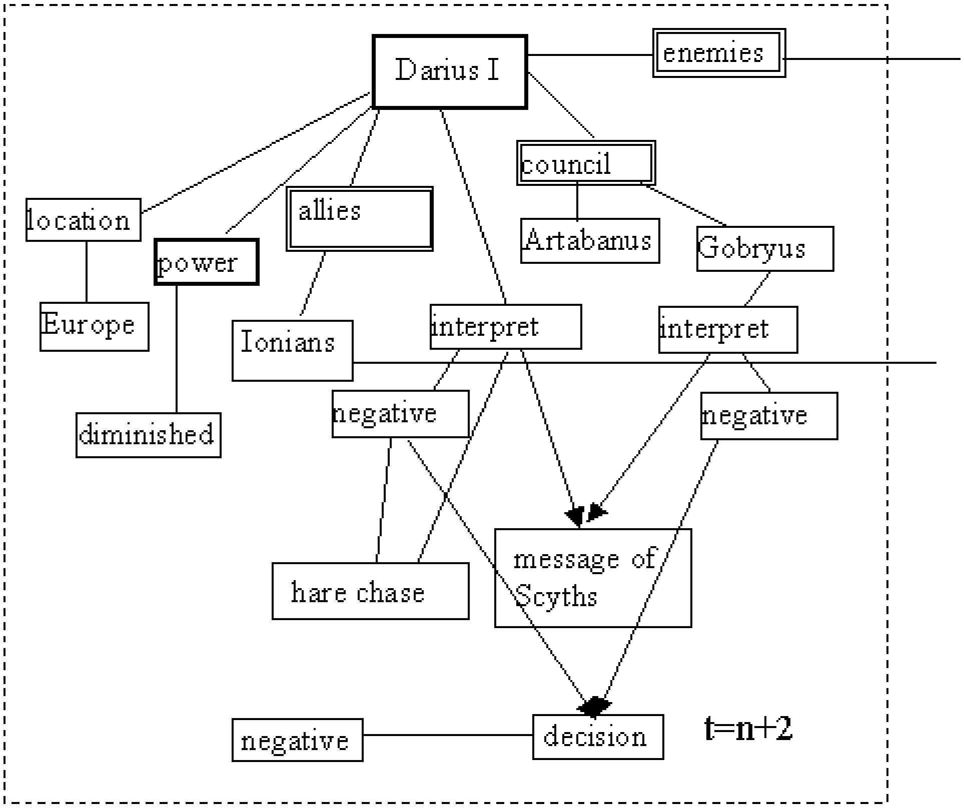


Figure 11. The final state of Darius configuration.

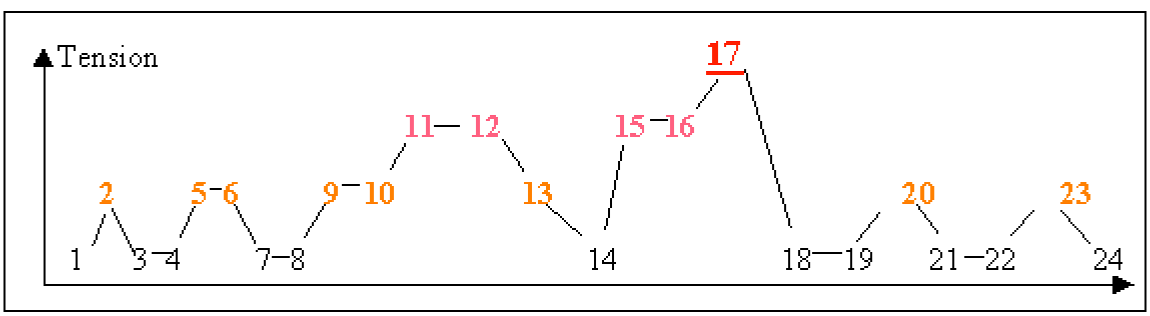


Figure 12. The tension landscape of the Darius' expedition. Point 17 marks a border between two components: march and return.

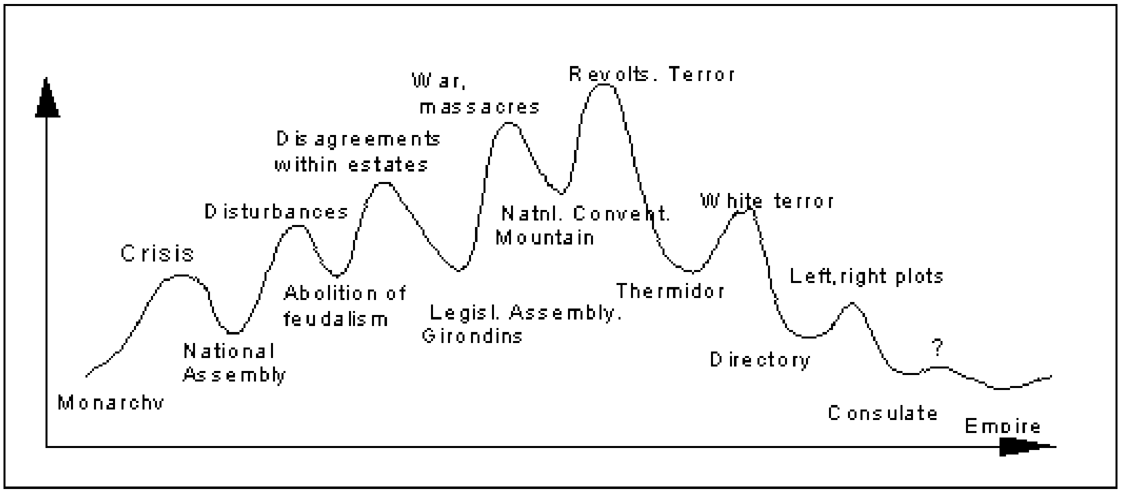


Figure 13. The tension profile of the French Revolution

CONCLUSION

As the historical sketch reveals, a mostly disconnected diaspora of natural scientists interested in history has been forming for some time around the legacy of Rashevsky, Richardson, Tilly, and others, starting from Democritus, Lucretius, and Leibniz.. Analytical history of Roehmer and Syme, as well as the study of patterns of military conflicts of Brecke, have a potential of becoming active centers of the “naturalization” of historical research.

A conundrum of the existing formal approaches to complex systems is the use of closed mathematical structures for representing open irreversible systems. Pattern Theory is suggested as another entry in the inventory of approaches to history. It could be a member of a future club of complementary concepts and methods.

Pattern Theory, with its atomistic realism, flexibility, and preservation of semantics, is uniquely positioned for developing a general representation and modeling of Very Complex Open Systems, such as life, mind, and society. The first point of the application of Pattern Theory to history can be transition state characterized by its irregularity.

Analytical history and large historical databases can create the medium where the ideas of Pattern Theory can be tested.

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